UNDERSTANDING THE OUTSTANDING: ZIPF'S LAW AND POSITIVE DEVIANTS

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When I entered college as a freshman 50 years ago, I did not imagine that even undergraduates could engage in scientific research. The opportunity to do so, which came in the summer after my sophomore year, was a transforming experience. Focusing on a project made much more meaningful the courses I'd had. Discovering that the frontiers of science were actually nearby was surprising and exhilarating. Reading research papers, and tracing back via footnotes the genealogy of ideas and techniques, greatly broadened my perspective. From that I began to appreciate what was outstanding work (as well as what was not so good). Also, it helped me to realize that first-rate scientific research does not stem so much from technical expertise as from architectural vision. Conversations with my faculty mentor and more senior student researchers taught me much, including important aspects of the culture and ethics of science. Discussions of concepts entirely new to me, particularly quantum physics, led me to see the pervasive power of mathematics. Consequently, I enrolled that Fall in a course in probability theory which led to many other unanticipated academic adventures that shaped my career. From this personal history, and what I have witnessed with many of my own students, I am convinced that undergraduate research often contributes a vital impetus to the making of a scientist.

The Academic Excellence study[1] of the role of research in science education at undergraduate institutions has compiled a large body of striking data. This reveals a wide range in quality of performance, even for institutions similar in function, organization, and resources. As with laboratory explorations, unorthodox means of analysis may prove useful in interpreting the data or devising new initiatives. In this essay, I call attention to two approaches that have emerged from empirical studies of complex phenomena and deserve to be better known. Although exemplified in distant contexts, these offer compelling perspectives on generic aspects of exceptional performance.

First is Zipf's Law, originally discovered in the form of an inverse correlation between word rank and word frequency in languages but found applicable as well to many other things, including population patterns, income distributions, and the severity of storms. These correlations appear akin to "pink noise," also termed "1/f noise," an inverse relation between spectral intensity and frequency. It is widely observed in acoustic science and invoked in the aesthetics of music. This

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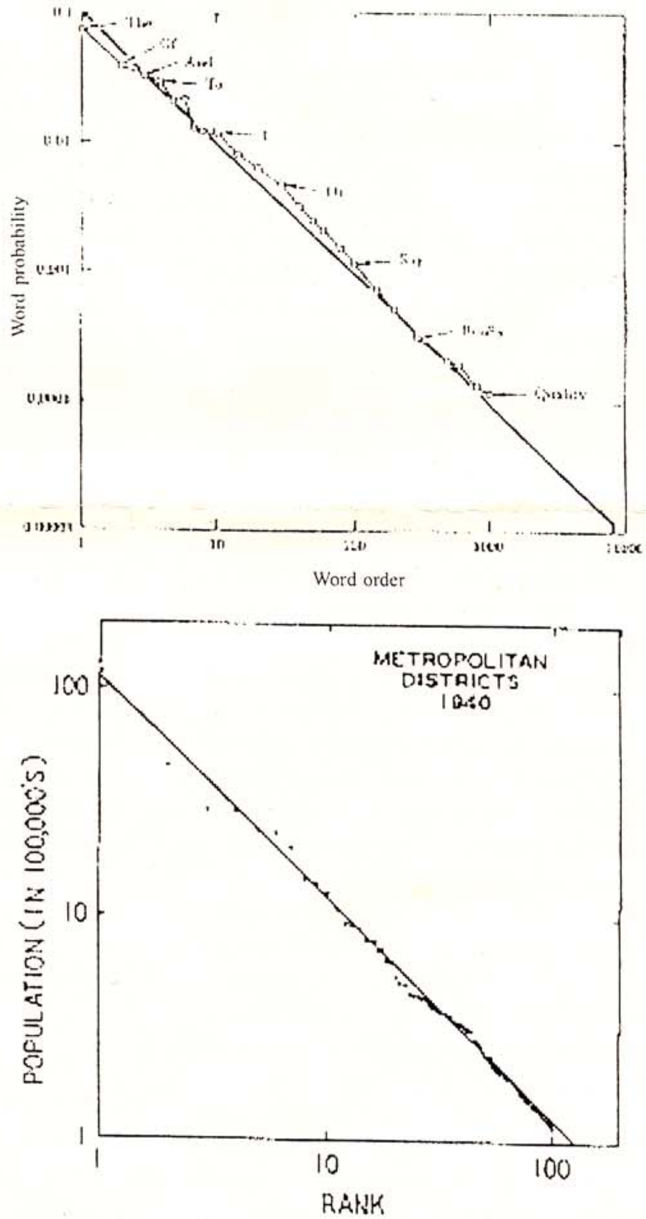
behavior, as interpreted by complexity theory, offers a qualitative paradigm for scientific productivity; it emphasizes that each of many factors has to be favorable to attain outstanding performance.

Beyond assessment looms the challenge, beset by familiar obstacles, of accomplishing significant and lasting change in institutions grown accustomed to modest or miserable performance. A strategy for such reform focuses on "positive deviants." This involves identifying exceptional people or practices that are already successful within an organization or community and devising means to amplify their impact. Inducing in this way change "from within rather than from without" has proved remarkably effective in reducing malnutrition of children in Vietnam and other poor countries. The same approach is now being increasingly applied to contend with other daunting societal problems as well as to enhance the performance of manufacturing or marketing firms.

ZIPF'S LAW AND PINK NOISE

George Kingsley Zipf (1902-1950), a professor of linguistics at Harvard, became intrigued by what he termed the "psycho-biology of language." This stemmed from discovering that if he ranked the different words in a given text by how often they were used, their frequency was approximately inversely proportional to the rank. Thus, the relative frequencies of the highest-ranking English words (*the, of, and, to,. . .*) are approximately 1, 1/2, 1/3, 1/4, . . . respectively. Likewise, if one word appears a thousand times in the text, ten words appear about a hundred times, a hundred words appear about ten times, and about a thousand words each occur just once.

Zipf devoted more than twenty years to examining similar correlations for many languages and for much other data, ranging from sizes of populations and



Examples of "Zipf's Law," illustrating rank-frequency distribution of English words (upper panel, from ref.6) and of populations of the one hundred largest metropolitan districts in the United States in 1940 (lower panel, from ref.2).

economic activities to the length of speeches in plays. He attempted to rationalize all this in a book published in 1949, intended to establish a *Principle of Least Effort* which would "facilitate an exact science of living behavior" in emulation of physics.[2] Today such correlations are regarded as empirical, although complexity theory has elucidated aspects linked to fractals and chaos.[3]

The term "Zipf's Law," referring to events whose frequency is approximately inversely proportional their rank, is applied both to common events (with top rank assigned to the most frequent, as with words) and to rare events (with top rank assigned to the most infrequent, as with sizes of cities or storms). A recent compilation of articles and books pertaining to Zipf's Law lists over 150 items.[4] It does not include, however, a 1928 paper by Edward Condon, a physicist, which evidently anticipated Zipf's original analysis of word frequencies.[5]

In the related phenomenon of "1/f noise," observed in a wide variety of physical systems such as semiconductor devices, the noise power is inversely proportional to the frequency.[6] This is called "pink noise" because the frequency dependence is intermediate between that of "white noise" (which does not vary with frequency) and "brown noise" (proportional to the inverse square of the frequency). Pink noise is often used as a test signal in acoustic research, because it has equal power in octave frequency bands, a property that approximately mimics many naturally occurring sounds. Pink noise also excites an approximately constant density of acoustic nerve endings in our ears. Analysis has shown that in most music the spectra of semitone intervals between successive notes and their amplitudes can both be approximated as inversely proportional to frequency over a large range.[7] Composers seem intuitively to write "pink music," intermediate between "white," which would have successive notes entirely random, and "brown," which would render the notes too predictable.

According to complexity theory, pink noise or Zipf's Law behavior typically arises for events or processes that require the contribution of many independent variables. The demonstration[8] involves some arduous mathematics, but its qualitative essence is simple. As a chemist, I naturally suggest an analogy to the daunting task of optimizing the yield of a multistep chemical synthesis. The maximum yield of the final product, the top-ranked result, can only be achieved in one way: by obtaining the largest possible yield in each of the successive steps. However, there are many ways to get less than the largest possible yield in one or more of the steps! Increasingly lower-ranked results thus can be obtained in increasingly many ways. This ensures that the rank or size of the final yield and its frequency or likelihood are inversely related.

Charles Townes, a remarkably productive physicist, has pointed to Zipf's Law as an intrinsic aspect of scientific productivity.[9] He notes that some scientists publish far more papers or obtain far more patents than the average, even though they may differ only a little in IQ from others. Whereas IQ is distributed in accord with the familiar bell-shaped curve, it is only one of many factors involved in human productivity. "Each of these factors brings in its own bell-shaped curve, and the sum of the many bell-shaped curves results in a 1/f-type law." As a consequence, high-ranking achievements will rarely emerge unless sufficient support is forthcoming for the inevitably far more numerous efforts that yield lesser results. To ensure good productivity, each of many factors has to be favorable. Society must value new ideas and discoveries; accept long-range prospects; foster diversity in approaches and institutions; tolerate failures and encourage trial and error, as nobody "can plan what scientific research is going to be successful." These requisites have been affirmed in other ways, but Zipf's Law adds a perspective aptly rooted in complex and chaotic processes.

AMPLIFYING POSITIVE DEVIANCE

Save the Children, in response to an appeal from the government of Vietnam, undertook in the 1990s an intensive effort to reduce childhood malnutrition, endemic in the rural villages. In desperation, the staff members in charge, Jerry and Monique Sternin, decided to try an unconventional approach. I will simply quote from their story,[10] as it has the character of a parable.

"We first sought out very poor families who had managed to avoid malnutrition. Although the parents in those families had access to no more resources than their neighbors, they somehow found enough food to keep their children healthy. By examining the behavior of these people, the positive deviants in the community, we hoped to find local strategies for combating malnutrition."

"It turned out that the mothers in those families were going out every day to nearby rice paddies and collecting tiny shrimps and crabs, which they were adding, along with sweet-potato greens, to their children's meals. They were also feeding their children three or four times a day, rather than the customary twice a day. The shellfish and greens were both readily available and free for the taking, but the conventional village wisdom held these foods to be inappropriate for young children. It was clear, therefore, that the immediate solution to the malnutrition problem did not require a lot of money or other outside resources; it simply required the community members to change their behavior and to start emulating the positive deviants in their midst."

"We launched a program to demonstrate to all the mothers the value of shellfish and greens and frequent feeding. For the first two weeks . . . the participating mothers were required to forage . . . and to bring a supply to the daily sessions, where they learned to cook them; [soon they] could see that the new foods did not make their children ill [but rather that the children were] becoming healthier. Within two years, 80% of the children participating in the project were no longer malnourished."

The project, begun in a few villages, was extended to many others in 20 provinces, in each case identifying local foods that had been employed by the positive deviants, such as peanuts, sesame seeds, snails, or dried fish. Save the Children has since used the approach in several other developing countries, with major impact on the lives of many millions of people. Other development organizations have adopted it to "address problems as various as ethnic conflicts and the spread of AIDS."

Sternin points out that "Most corporate change efforts resemble traditional development efforts: they focus on defining the organization's needs, and then they try to fulfill those needs by introducing resources and "best practices" from the outside. Often, though, the members of the organization resist the external solutions, and the desired performance gains prove either unattainable or fleeting." Now, however, companies and management-consulting groups are beginning to consider positive deviance to identify ways to improve performance and working conditions. Sternin[11] cites the case of a pharmaceutical company:

"[It had believed] that the more reps you had and the more calls you made on customers, the more you would sell. However, the positive deviants within the company, the most successful units, had fewer salespeople [and] made only one-third the usual number of visits to customers per day. . . . These reps were spending far more time with individual doctors, educating them on the benefits and uses of the products . . . and they were outselling the others by a big margin."

The strategy of amplifying positive deviants is itself in accord with the pragmatic and "hands-on" precepts of scientific research. It emphasizes the key role of "outliers," whose example can empower a community or institution to achieve performance well beyond presumed limitations.

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[1] Michael P. Doyle, Ed., *Academic Excellence* (Tucson, Ariz.: Research Corporation, 2000).

[2] George Kingsley Zipf, *Human Behavior and the Principle of Least Effort* (Cambridge, Mass.: Addison-Wesley Press, 1949).

[3] Murray Gell-Mann, *The Quark and the Jaguar* (San Francisco: W.H. Freeman and Co., 1994): 92–98.

[4] Wentian Li, "References on Zipf's Law," on web at http://www.linkage.rockefeller. edu/wli/zipf/.

[5] Edward U. Condon, "Statistics of Vocabulary," *Science* 67 (1928): 300.

[6] Manfred Schroeder, *Fractals, Chaos, and Power Laws* (San Francisco: W.H. Freeman and Co., 1991): 33–38, 107–112, 122–126.

[7] R.V. Voss and J. Clark, "1/f Noise in Music; Music from 1/f Noise, *Journal of the Acoustical Society* 63 (1978): 258.

[8] Elliot W. Montroll and Michael F. Shlesinger, "Maximum Entropy Formalism, Fractals, Scaling Phenomena, and 1/f Noise: A Tale of Tails," *Journal of Statistical Physics* 32 (1983): 209.

[9] Charles H. Townes, "Unpredictability in Science and Technology," in *Science and Society*, Martin Moscovits, Ed. (Concord, Ontario: House of Anansi Press, 1995): 31–42.

[10] Jerry Sternin and Robert Choo, "The Power of Positive Deviancy," *Harvard Business Review* 78 (January 2000): 14.

[11] David Dorsey, "Positive Deviant," *Fast Company* 41 (December 2000): 284; on web at http://www.fastcompany.com/online/41/sternin.html.